Workshop Exotic Hadrons: Theory and Experiment at Lepton and Hadron Colliders

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Tetraquarks at large N_c

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Multiquark states in QCD

Possibility of existence multiquark states considered long ago by many authors (Jaffe, 1977).

However, theoretical difficulties arise in QCD.

QCD is a confining theory, where color gauge invariance plays a crucial role. The question is: Can QCD produce by the sole confining forces bound states of multiquarks, containing more than a pair of valence quark-antiquarks for mesons and more than three valence quarks for baryons?

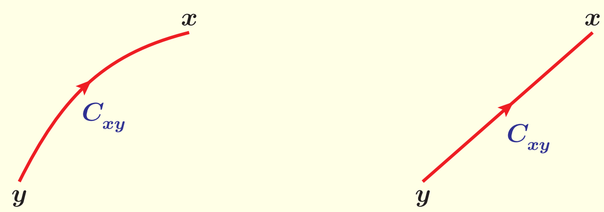
Molecular type multiquark states, which result from the direct interaction of ordinary hadrons, do not raise this conceptual problem, since the mutual interaction of ordinary hadrons is no longer confining.

It is generally admitted that observable quantities should be color gauge invariant; confinement operates between color non-singlet quantities.

To construct, with quark fields, gauge invariant operators, one needs the introduction of path-ordered gluon field phase factors (also called Wilson lines), which play the role of color parallel transporters:

$$U^a_b(C_{xy}) = \left(Pe^{-ig\int_{C_{xy}} dz^\mu A^B_\mu(z)T^B}\right)^a_b,$$

where C_{xy} is an oriented line going from y to x.

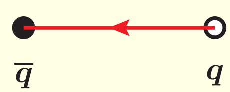


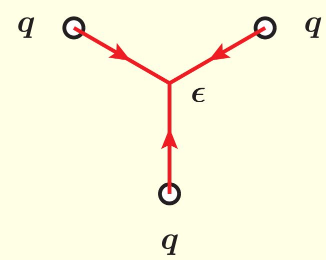
For mesonic and baryonic gauge invariant operators, one has the following constructions:

$$M = \overline{q}_a(x)U^a_b(C_{xy})q^b(y),$$

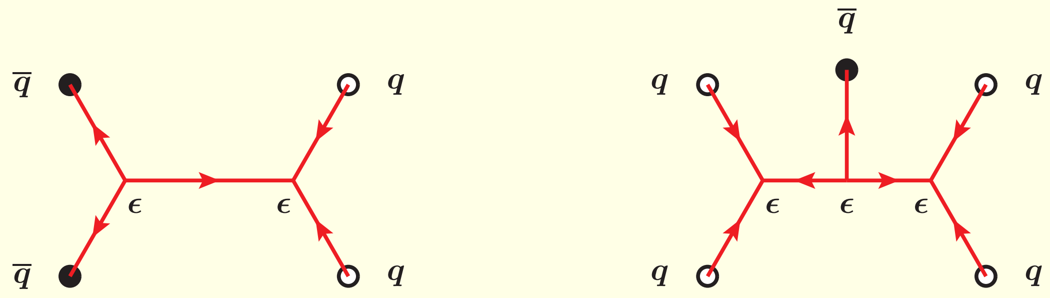
$$B = \varepsilon_{abc}U^a_d(C_{xy})q^d(y)U^b_e(C_{xt})q^e(t)U^c_f(C_{xz})q^f(z),$$

(quark flavor and spin indices omitted) with a corresponding pictorial representation





Similarly, multiquark color gauge invariant operators can be constructed by means of the path-ordered phase factors. For tetraquarks and pentaquarks, one has the following constructions:



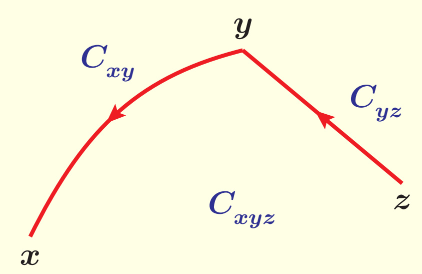
Problem: These operators are not color irreducible. They can be decomposed into combinations of products of ordinary mesonic and baryonic operators.

Color reducibility of multiquark operators

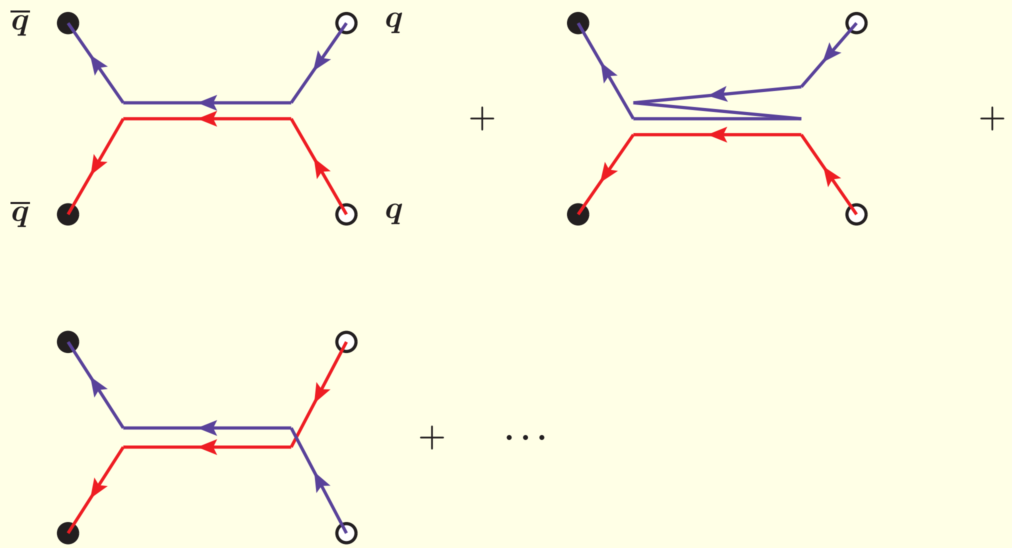
Property already known with local color-singlet operators, by means of Fierz transformations. It remains true also with multilocal operators. (Jan Stern, 1980.)

Basic ingredient: Path-ordered phase factors are elements of the color gauge group $SU(3)$. They satisfy:

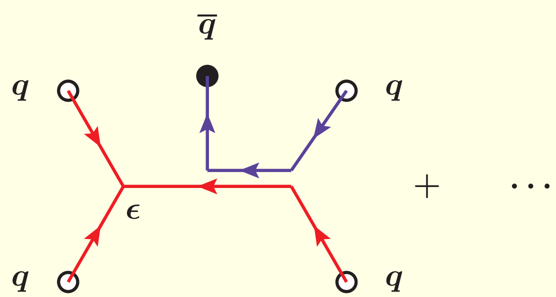
$$\det(U) = 1, \qquad U^a_b(C_{xy})U^b_c(C_{yz}) = U^a_c(C_{xyz}).$$



Tetraquark operators are decomposed as follows:



Pentaquark operators:

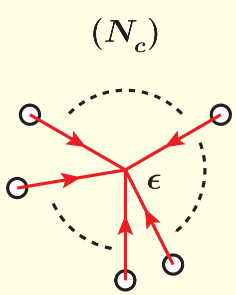


Therefore, multiquark states cannot be on the same footing as ordinary hadron states. Since mesonic and baryonic operators are color gauge invariant, their mutual interactions are expected to be nonconfining. If multiquark operators are generally decomposable along products of mesonic and/or baryonic operators, one should not expect that multiquark states are confined states. At most they would be molecular type bound states or resonances.

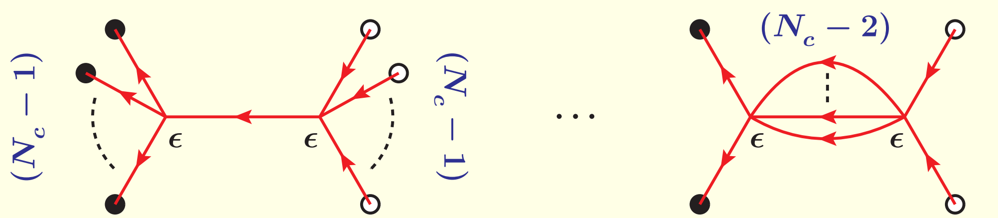
One way to evade this outcome: existence of a (time dependent?) fine tuning dynamical mechanism, which prevents the previous decompositions. It tightens, in the tetraquarks for example, the two quarks together and the two antiquarks together, to form with them almost pointlike objects that produce, as a whole, a confined bound state. Diquark model. (Jaffe and Wilczek, 2003; Shuryak and Zahed, 2004; Maiani, Piccinini, Polosa and Riquer, 2005.) Dynamical mechanisms proposed by Brodsky, Hwang and Lebed (2014) and Maiani, Polosa and Riquer (2018).

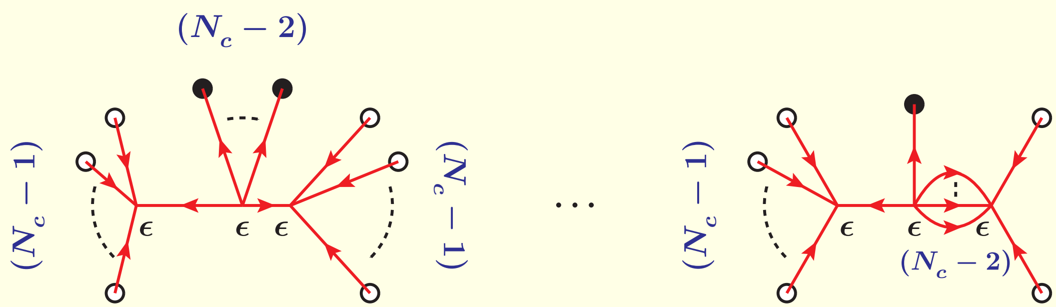
QCD at large N_c

Assuming the possible existence of such a mechanism, one might analyze in more detail its consequences. For this, we will have recourse to the large-N_c behavior of QCD. This is motivated by the fact that QCD, as a non-Abelian gauge theory, can be continued parametrically to the color gauge group $SU(N_c)$ with respect to the parameter N_c ('t Hooft, 1974; Witten, 1979). At large N_c, many nonperturbative aspects of the theory simplify, due to the topological properties of Feynman diagrams. In particular, inelasticity effects become nondominant. In the large-N_c limit, mesons become stable noninteracting particles, with finite masses, while baryons have masses increasing with N_c.



Structure of tetraquarks and pentaquarks at large N_c.





The color reducibility property of multiquark operators remains also valid for general N_c.

The tetraquark operator decomposes into combinations of products of mesonic operators and eventually of Wilson loops.

The pentaquark operator decomposes into combinations of products of mesonic operators and one baryonic operator and eventually of Wilson loops.

Because of the above general reducibility phenomenon of multiquark operators, it is preferable to continue the study of the tetraquark problem in the sector of two mesonic operators, at any N_c.

Properties of tetraquarks at large N_c

Tetraquark operator:

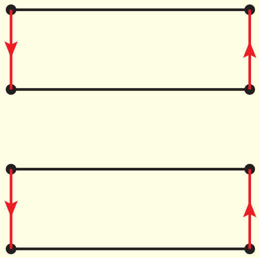
$$T(x,y;t,z) = M(x,y)M(t,z),$$

$$M(x,y) = \overline{q}_a(x)U^a_b(C_{xy})q^b(y),$$

quark flavor and spin indices omitted. For simplicity, four different quark flavors are considered.

Calculate the correlation function of two Ts. The general counting rules of large-N_c behavior are not modified: The path-ordered phase factors transport color in the fundamental representation.

$$\langle\, T(x,y;t,z)\; T^{\dagger}(x',y';t',z')\,\rangle =$$



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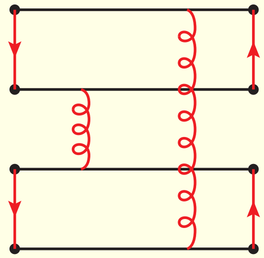
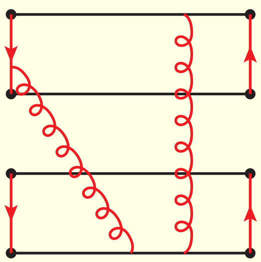


$O(N_c^2)$ $\qquad$ $O(N_c^0)$

$+$



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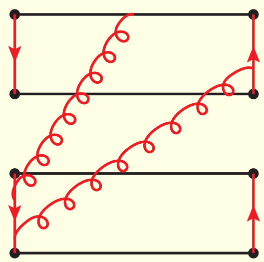
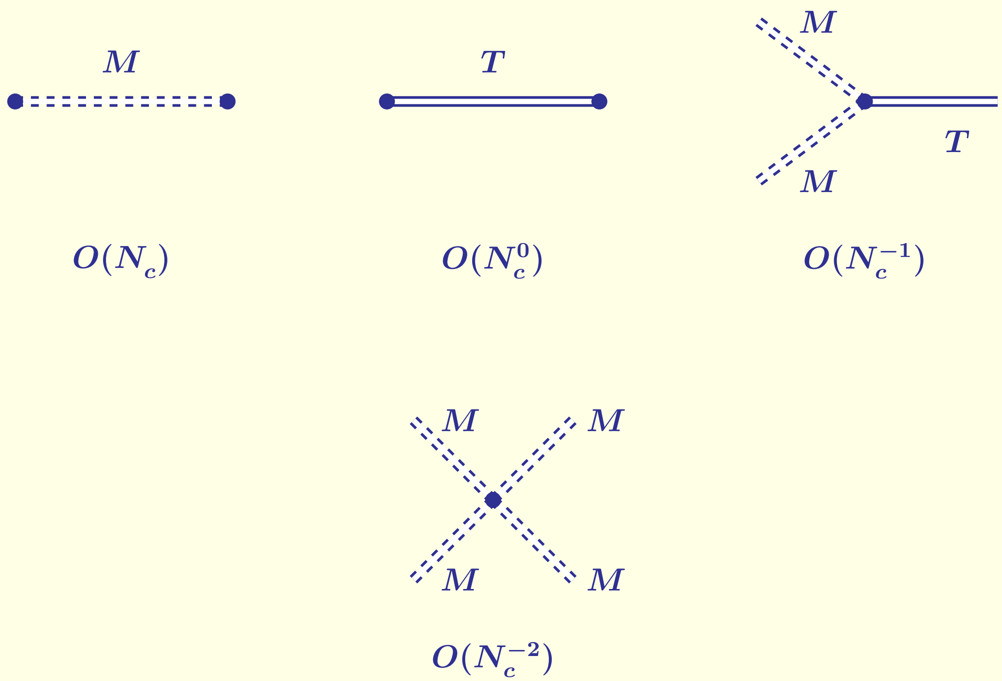


$+ \quad \cdots$

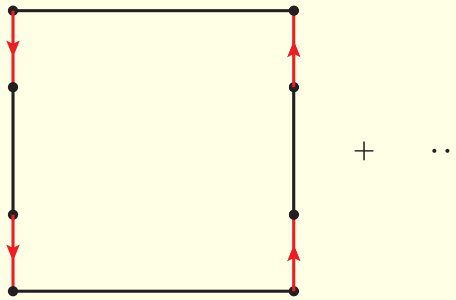
$O(N_c^0)$ $\qquad$ $O(N_c^0)$

The correlation function is domainated at large N_c by the propagation of two free mesons. Confirms Coleman's conclusion (1979), obtained with local tetraquark currents. Tetraquarks may appear only in nonleading diagrams, containing, in the s-channel, two quark and two antiquark lines.

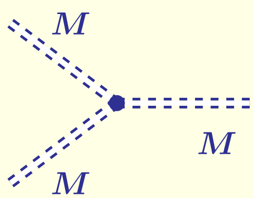
From these diagrams, one also extracts the large-N_c behaviors of the meson and tetraquark propagators, as well as tetraquark-meson and meson-meson couplings.



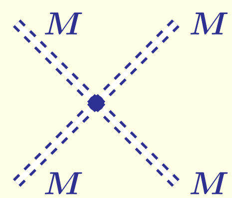
The four-meson coupling is here of order N_c^{-2}, but is specific to the case of four different quark flavors. The generic behavior, $O(N_c^{-1})$, corresponds to the cases where one has three or two different quark flavors, where additional Feynman diagrams occur.



$O(N_c)$



$O(N_c^{-1/2})$



$O(N_c^{-1})$

Some features of the diquark model can also be deduced from the comparison of the quadratic Casimirs of the diquark system (in its antisymmetric representation) and of the quark-antiquark system: the former is equal to $1/(N_c - 1)$ times the latter. Similarly for the antidiquark system. The quadratic Casimir of the diquark-antidiquark system is of the same order as that of the quark-antiquark system: $2(N_c - 2)/(N_c - 1)$. (Esposito, Pilloni,Polosa, 2017.)

At large N_c, the diquark energy spectrum, could be determined, in a nonrelativistic approximation, by a Schrödinger equation of the following type:

$$\Big[E - \frac{\mathbf{p}^2}{2\mu} - \frac{1}{N_c}\sigma r\Big]\phi = 0.$$

(μ: the reduced mass of the two quarks composing the diquark; σ: the string tension.)

The diquark bound state energy and its spatial size scale as

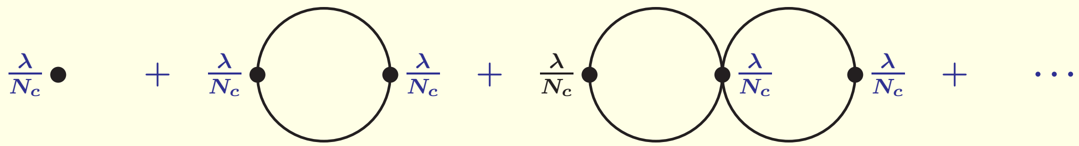
$$E \sim \Big(\frac{N_c\sqrt{\sigma}}{2\mu}\Big)^{1/3}\frac{\sqrt{\sigma}}{N_c}, \qquad < r > \sim \Big(\frac{N_c\sqrt{\sigma}}{2\mu}\Big)^{1/3}\frac{1}{\sqrt{\sigma}}.$$

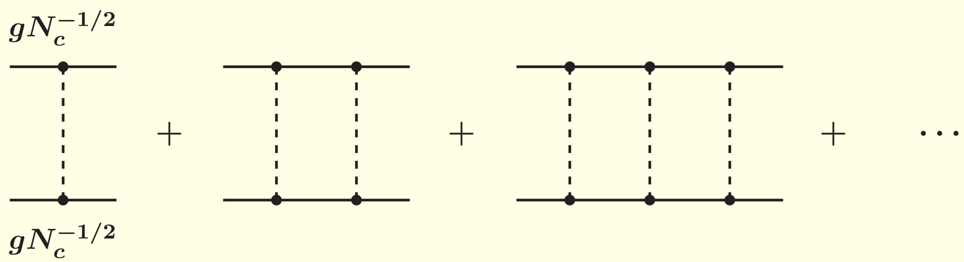
At large N_c, the spatial size increases (weakly) and the diquark ceases to be a compact object. In parallel, its energy decreases and the diquark mass tends to the two-quark mass threshold.

The large-N_c limit does not seem to be favorable for diquark formation.

Molecular type tetraquarks

Study by means of meson-meson scattering amplitude. Resonances or bound states are produced through sums of chains of bubble and box diagrams of effective meson-meson interactions.





The sums involve contributions of decreasing orders of N_c. This is in contrast to the case of compact tetraquarks, where the tetraquark poles are assumed to be produced within a definite order of N_c contributions, because they are produced from a direct interaction of quarks and gluons.

For light mesons, use of chiral perturbation theory allows a precise determination of the four-meson couplings at low energies and a coherent summation of the chain of bubble diagrams with an extension of the domain of energies with the aid of elastic unitarity and dispersion relations. One obtains a scalar resonance with mass and width behaving at large N_c as

$$M \sim \sqrt{N_c}, \qquad \Gamma \sim \sqrt{N_c}.$$

(Peláez and Rios, 2006; Peláez, 2016.)

For heavy mesons, a nonrelativistic approximation can be used. For bound states, Yukawa-type interactions (exchange of light mesons), with the chain of box diagrams, can be used. Bound states exist only if $g^2/N_c > \gamma_0$. $\Longrightarrow$ For $N_c \to \infty$, the bound states disappear. For values of N_c satisfying the inequality, at least one bound state exists with a mass approaching the two-meson threshold. (Luke and Manohar, 1997.)

The chain of bubble diagrams also produces a bound state (if $\lambda < 0$), with binding energy

$$E \sim -\frac{N_c^2}{\lambda^2}.$$

(Weinberg, 1991; Luke and Manohar, 1997.) When $N_c \to \infty$, $E \to -\infty$ and the bound state disappears from the bottom.

In general, when $N_c \to \infty$, bound states and resonances disappear from the spectrum. Not surprising, since in that limit, mesons do not interact.

Reviews (non-exhaustive): Amsler and Törnqvist, 2004; Swanson, 2006; Chen, Chen, Liu and Zhu, 2016; Hosaka, Iijima *et al.*, 2016; Esposito, Pilloni and Polosa, 2017; Ali, Lange and Stone, 2017; Lebed, Mitchell and Swanson, 2017; Guo, Hanhart *et al.*, 2018; Olsen, Skwarnicki and Zieminska, 2018; Karliner, Rosner, Skwarnicki, 2018.

Conclusion

The large-N_c limit of QCD allows us to have a qualitative insight about the possible structure of multiquark states.

Gauge invariant operators describing these states are color reducible and therefore multiquark states do not lie on the same footing as ordinary mesons and baryons.

In the $N_c \to \infty$ limit, they disappear from the particle spectrum, whether they are compact or molecular. Finite values of N_c may accomodate their possible existence.

Compact tetraquarks, generated by the diquark mechanism, need additional favorable circumstances at their experimental production stage to be created. If so, they have the tendancy to be accumulated near the two-meson thresholds with small decay widths, due mainly to their small couplings to two mesons.

Molecular tetraquarks are produced as resonances at large mass values with large decay widths. As bound states, they may be created near the two-meson thresholds.