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On the role of dynamical quark mass generation in chiral symmetry breaking in QCD

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Framework

Gauge invariant investigation of quark dynamics in QCD.
Gauge invariant objects expected to provide a more precise description of physical observables. Better infrared behavior, free of spurious singularities.

Necessity of introducing gluon field path-ordered phase factors (parallel transport operators) to ensure gauge invariance.

Paths along polygonal lines are of particular interest, since they can be decomposed into a succession of straight line segments.

$U(y,x)$ denotes a path-ordered phase factor along the straight line segment going from x to y.

For the quark Green's functions the polygonal lines form a complete set of paths.

Reference: arXiv, 1304.0961, P.R. D 88 (2013) 025034.

Gauge invariant Green's functions along polygonal lines

Quark gauge invariant Green's functions with polygonal lines can be classified according to the number of segments they contain.

The gauge invariant two-point quark Green's function with a polygonal line with n segments and $(n-1)$ junction points y_1, y_2, ...,y_{n-1} between the segments is defined as

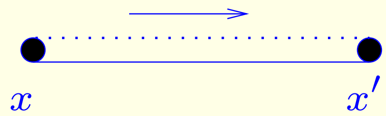
$$S_{(n)}(x,x';y_{n-1},\ldots,y_1) = -\frac{1}{N_c}\,\langle\overline{\psi}(x')U(x',y_{n-1})\ldots U(y_1,x)\psi(x)\rangle,$$

where each U is along a straight line segment.

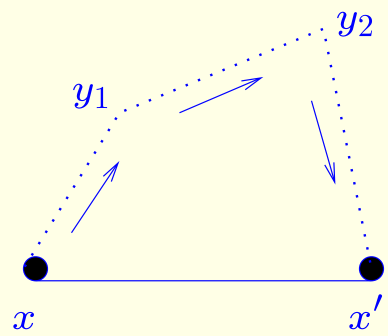
For one straight line, one has:

$$S_{(1)}(x,x') \equiv S(x,x') = -\frac{1}{N_c}\,\langle\overline{\psi}(x')\,U(x',x)\,\psi(x)\rangle.$$

Pictorially:



$$S(x,x') \equiv S_{(1)}(x,x') = -\frac{1}{N_c} < \overline{\psi}(x')\, U(x',x)\, \psi(x) >$$



$$S_{(3)}(x,x';y_2,y_1) = -\frac{1}{N_c} < \overline{\psi}(x')\, U(x',y_2)U(y_2,y_1)U(y_1,x)\, \psi(x) >$$

Integrodifferential equation for the two-point function

Use of equations of motion of Green's functions yield functional relations between the Green's functions of different classes of polygonal line.

$\Longrightarrow$ S is the only dynamically independent gauge invariant quark Green's function. All $S_{(n)}$s with $n > 1$ are calculable from $S_{(1)} \equiv S$.

One establishes the following integrodifferential equation for the Green's function $S(x, x')$ (the analog of the self-energy Dyson-Schwinger equation):

$$(i\gamma.\partial_{(x)} - m)S(x, x') = i\delta^4(x - x') + i\gamma^\mu \Big\{ K_{2\mu}(x', x, y_1)\, S_{(2)}(y_1, x'; x)$$

$$+ \sum_{n=3}^{\infty} K_{n\mu}(x', x, y_1, \ldots, y_{n-1})\, S_{(n)}(y_{n-1}, x'; x, y_1, \ldots, y_{n-2}) \Big\}.$$

The kernel K_n contains globally Wilson loops with polygonal contours having at most $(n + 1)$ sides and n functional derivatives on different sides.

Bound state wave functions

Two quark fields ψ_1 and ψ_2 with masses m_1 and m_2, respectively. Bound state with total momentum P. Wave functions:

$$\Phi_{(1)}(P, x_1, x_2) = \frac{-1}{\sqrt{N_c}} \langle 0|T\,\overline{\psi}_2(x_2)U(x_2, x_1)\psi_1(x_1)|P\rangle,$$

$$\Phi_{(n)}(P, x_1, x_2; y_{n-1}, \ldots, y_2, y_1) = \frac{-1}{\sqrt{N_c}} \langle 0|T\overline{\psi}_2(x_2)U(x_2, y_{n-1})U(y_{n-1}, y_{n-2})$$
$$\times \ldots U(y_2, y_1)U(y_1, x_1)\psi_1(x_1)|P\rangle.$$

Functional relations are established between $\Phi_{(n)}$, $\Phi_{(1)}$ and Wilson loops, similar to those of the two-point Green's functions.

Wave equations

One deduces from the equations of the four-point gauge invariant Green's functions the corresponding bound state wave equations.

$$
\begin{aligned}
(i\gamma.\partial_{x_1} - m_1)\Phi_{(1)}(P, x_1, x_2) = i\gamma^{\mu}\Big\{ \sum_{n=2}^{\infty} K_{1,n\mu} * \Phi_{(n)} \\
+ \quad \sum_{n=2}^{\infty} (N_{1,n\mu} * \Phi_{(1)}) * S_{2(n)} \Big\}.
\end{aligned}
$$

$$
\begin{aligned}
\Phi_{(1)}(P, x_1, x_2)(-i\gamma.\overleftarrow{\partial}_{x_2} - m_2) = -i\Big\{ \sum_{n=2}^{\infty} \Phi_{(n)} * K_{2,n\nu} \\
+ \quad \sum_{n=2}^{\infty} S_{1(n)} * (\Phi_{(1)} * N_{2,n\nu}) \Big\}\gamma^{\nu}.
\end{aligned}
$$

Chiral symmetry breaking

A mechanism of chiral symmetry breaking is provided by the dynamical mass generaion phenomenon.
Nambu and Jona-Lasinio (1961);
Baker, Johnson and Lee (1964).

Models in QCD have been considered in the Coulomb gauge.
Finger, Mandula and Weyers (1980); Le Yaouanc et al. (1983); Adler and Davis (1984); Alkofer and Amundsen (1988); Lagaë (1992).

We show in the present gauge invariant formalism that the dynamical mass generation phenomenon provides a firm critereon for chiral symmetry breaking.

Consider the two-point gauge invariant Green's function $S_{(1)} = S(x_1, x_2)$ (with one straight line segment for the phase factor). It can be decomposed into a vector and a scalar parts:

$$S(x) = i\gamma.\partial F_1(x^2) + F_0(x^2).$$

In perturbation theory, the scalar component F_0 vanishes when the quark mass m goes to zero.

If the solution of the equation of S yields a scalar part that does not vanish in the limit of a massless quark, then one is in the presence of a nonperturbative solution which may be described as a phenomenon of dynamical mass generation, although the structure of this mass term might be complicated (not a simple pole).

The equation satisfied by F_0 can be extracted from that of S by calculating the anticommutator of S with γ_5. In the limit of massless quarks, one obtains:

$$i\gamma.\partial_{(x_1)}[\gamma_5, S_{(1)}]_+ = i\gamma^\mu \Big\{ \sum_{n=2}^{\infty} K_{n\mu} * [\gamma_5, S_{(n)}]_+$$

$$+ \quad \sum_{n=2}^{\infty} (N_{n\mu} * [\gamma_5, S_{(1)}]_+) * S_{(n)} \Big\}.$$

Equation to be compared with that of the wave function Φ in the massless limit:

$$i\gamma.\partial_{x_1}\Phi_{(1)}(P, x_1, x_2) = i\gamma^\mu \Big\{ \sum_{n=2}^{\infty} K_{n\mu} * \Phi_{(n)}$$

$$+ \quad \sum_{n=2}^{\infty} (N_{n\mu} * \Phi_{(1)}) * S_{(n)} \Big\}.$$

(Quark indices 1 and 2 have been removed, since the quarks are massless.)

Similar equations are also established for $[\gamma_5, S_{(n)}]_+$, where $S_{(n)}$ is the Green's function with a polygonal line with n sides, and compared with the equation of $\Phi_{(n)}$.

We observe that the equation of $[\gamma_5, S_{(1)}]_+$ is the same as the equation of $\Phi_{(1)}$, provided the following correspondences are made: $[\gamma_5, S_{(n)}]_+ \longrightarrow \Phi_{(n)}, n = 1, 2, \ldots$.

Therefore, if the equation of $S_{(1)}$ has, in the massless quark limit, a nonvanishing solution for $[\gamma_5, S_{(1)}]_+$, then this is also a solution of the bound state equation for a pseudoscalar state. However, in general, the wave function $\Phi_{(1)}$ has also a dependence on the total momentum P of the state, but the latter variable does not exist in $S_{(1)}$. Therefore the matching is possible only if $P = 0$, which means that $P^2 = 0$: the corresponding pseudoscalar state is massless and is a Goldstone boson for chiral symmetry breaking.

Comments

1) The above results are exact statements of QCD, although for unrenormalized quantities. All expressions of the kernels are explicitly known and no approximations were made throughout the calculations.

2) One may adopt at the practical level of resolution of the equations an approximation scheme based on the truncation of the series of kernels according to the number of sides of the polygonal contours of the Wilson loops (the first one beginnining with a triangular contour). Using the same type of truncation in both types of equation (Green's function and wave function), one still has the same conclusion as in the exact case. Therefore, the relationship between dynamical mass generation and chiral symmetry breaking remains stable under consistent approximation schemes.

3) The conclusions that are drawn are gauge invariant, since all quantities in consideration are gauge invariant and this feature is not altered by the approximation scheme of the truncation type.

Two-dimensional QCD

Many simplifications in two-dimensional QCD at large N_c.

The equations of the Green's functions can be solved exactly and analytically. The solutions are infrared finite with singularities lying on the positive real axis of p^2 (timelike region). They are represented by an infinite number of branch points, characterized by positive masses M_n $(n = 1, 2, \ldots)$. They are stronger than simple poles and this feature might prevent observability of quarks as free particles.

The quark condensate is different from zero.

The threshold masses M_n represent dynamically generated masses and the scalar part of the Green's function yields a solution of the bound state equation with zero mass and zero total momentum.