Large-N_c QCD and tetraquarks
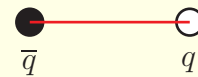
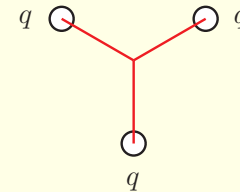Hagop Sazdjian

IPN Orsay, Univ. Paris-Sud, Univ. Paris-Saclay

Multiquark states in QCD

Hadrons are color-singlet bound states of quarks and gluons.

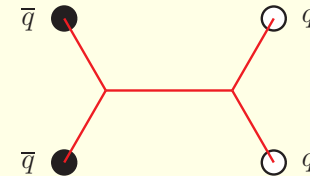Mesons are essentially made of $\overline{q}q$:
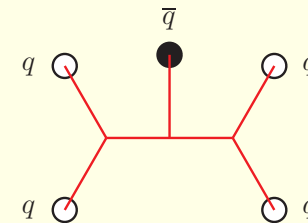
Baryons are essentially made of qqq:

Are there other types of structure for bound states (exotics)?

Tetraquarks would be made of $\overline{q}\overline{q}qq$:

Pentaquarks would be made of $\overline{q}qqqq$:

Possibility of multiquark states considered long ago by many authors (Jaffe, 1977).

However, theoretical difficulties arise in QCD.

We concentrate in the following on the tetraquark problem.

The difficulty is related to the fact that a tetraquark field, local or nonlocal, made of a pair of quark and a pair of antiquark fields, which would be color-gauge invariant, could be decomposed, by Fierz transformations, into a combination of products of color-singlet bilinear operators of quark-antiquark pairs.
For instance, in local form

$$T(x) = (\overline{q}\overline{q}qq)(x) \sim \sum (\overline{q}q)(x)\,(\overline{q}q)(x),$$

where $(\overline{q}q)(x)$ are themselves color-singlet.

However, color-singlet $(\overline{q}q)(x)$'s essentially describe ordinary meson fields or states. The above decomposition is suggestive of a property that tetraquarks would be factorizable into independent mesons and could at best be bound states or resonances of mesons, called also molecular tetraquarks, and not genuine bound states of two quarks and two antiquarks, resulting from the direct confinement of the four constituents.

What would be, on phenomenological grounds, the difference of the two types of bound state, since both of them would be represented by poles in the hadronic sector?

Meson-meson interaction forces are short-range and weak, as compared to the strong long-distance confining forces.

Therefore, molecular type tetraquarks would be loosely bound states, with relatively large space extensions, while tetraquarks, formed directly by confining forces, would be more tightly bound. The latter are called compact tetraquarks.

Compact tetraquarks would also exist in flavor multiplicities, since confinement is independent of flavor.

The above qualitative differences have their influence on phenomenological quantities, like the number of states, decay modes, decay widths and transition amplitudes.

For more than ten years, many tetraquark candidate states have been signalled by several experiments: Belle, BaBar, BESIII, LHCb, CDF, D0, CMS. Ordinary meson structures could not fit their properties.

Intense theoretical activity around the extraction of their physical properties and their interpretation. However, difficulties to explain all data by a single model or mechanism.

Many review articles.

T. Cohen *et al.* (2014); S.L. Olsen (2015); H.-X. Chen *et al.* (2016);
A. Hosaka *et al.* (2016); A. Esposito *et al.* (2016); M. Karliner (2016);
R.F. Lebed *et al.* (2017); F.-K. Guo *et al.* (2017); A. Ali *et al.* (2017);
S.L. Olsen *et al.* (2017).

Recent lattice results

No evidence

- HAL QCD collaboration, Ikeda *et al.* (2014), (2016). $\overline{u}\,\overline{d}cc$, $\overline{u}\,\overline{d}cs$, $\overline{c}\,\overline{d}cu$.

- Padmanath *et al.* (2015). $\overline{c}\,\overline{c}ud$, $\overline{c}c(\overline{u}u + \overline{d}d)$, $\overline{c}\,\overline{c}ss$.

- Hadron Spectrum collaboration, Cheung *et al.* (2017). $cc\overline{l}\,\overline{l}$, $cc\overline{l}\,\overline{s}$, $\overline{c}c\overline{l}l$.

- Hughes *et al.* (2017). $\overline{b}\,\overline{b}bb$.

Evidence

- Francis *et al.* (2017). $\overline{b}\,\overline{b}ud$, $\overline{b}\,\overline{b}ls$.

- Bicudo *et al.* (2017). $\overline{b}\,\overline{b}ud$.

We will be interested here by the qualitative properties of compact tetraquarks, since the very existence of compact tetraquarks is intimately related to fundamental properties of QCD, not yet well understood. The existence of molecular type tetraquarks does not raise any conceptual difficulty.

The diquark model, proposed by Jaffe and Wilczek (2003), and Nussinov (2003), provides a mechanism to produce compact multiquark states. It has been applied to tetraquark phenomenology by Maiani *et al.* (2004-).

We will have recourse in our investigation to the large-N_c limit of QCD, which might give us complementary informations about the problem.

QCD at large N_c

Framework: $SU(N_c)$ gauge theory, with quarks in the fundamental representation, considered in the limit $N_c \to \infty$ with $g \sim 1/N_c^{1/2}$. ('t Hooft, 1974.)

In this limit, QCD catches the main properties of confinement, while being simplified with respect to secondary complications. $1/N_c$ plays the role of a perturbative parameter.

Properties of the theory analyzed by Witten (1979).

Analysis done with two-point, three-point and four-point functions of quark color-singlet bilinear operators (currents) $j(x) = (\overline{q}\Gamma q)(x)$ and the study of their large-N_c behavior.
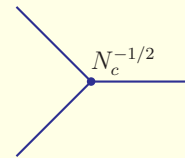
The spectrum is saturated by an infinite number of free stable mesons. (Infinite number dictated by asymptotic freedom.)
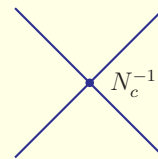
$$M_n = O(N_c^0).$$

Many-meson states contribute only to subleading orders.

Meson-meson interactions are weak:

- Three-meson couplings: $\sim N_c^{-1/2}$.

$N_c^{-1/2}$

- Four-meson couplings: $\sim N_c^{-1}$.

N_c^{-1}

Meson decay constants: $\langle 0|j_\mu|M(p)\rangle = p_\mu f_M, \quad f_M = O(N_c^{1/2}).$

Meson strong decay widths: $\Gamma(M) = O(N_c^{-1}).$

Tetraquarks at large N_c

Can we have similar predictions with tetraquarks?

$$T(x) = (\overline{q}\overline{q}qq)(x) \qquad (\text{color singlet}).$$

$$\langle T(x)T^\dagger(0)\rangle \underset{N_c\to\infty}{=} \langle j(x)j^\dagger(0)\rangle\,\langle j(x)j^\dagger(0)\rangle.$$

Equivalent to the propagation of two free mesons. (Coleman, 1980.)

No tetraquark poles can appear at this order.

For a long time, this fact has been considered as a theoretical proof of the non-existence of tetraquarks as elementary stable particles, surviving in the large-N_c limit, like the ordinary mesons.

Recently, Weinberg (2013) observed that if tetraquarks exist as bound states in the large-N_c limit with finite masses, even if they contribute to subleading diagrams, the crucial point is the qualitative property of their decay widths: are they broad or narrow? In the latter case, they might be observable. He showed that, generally, they should be narrow, with decay widths of the order of $1/N_c$, which is compatible with the stability assumption in the large-N_c limit.

Knecht and Peris (2013) showed that in a particular exotic channel, tetraquarks should even be narrower, with decay widths of the order of $1/N_c^2$.

Cohen and Lebed (2014) showed, for more general exotic channels, with an analysis based on the analyticity properties of two-meson scattering amplitudes, that the decay widths should indeed be of the order of $1/N_c^2$.

Lucha, Melikhov and H.S. (2017) show, on the basis of the singularity analysis of Feynman diagrams with the Landau equations, that tetraquark two-meson decay widths of order $1/N_c^2$ represent the most general case. For fully exotic tetraquarks (four different quark flavors) two different tetraquarks, each having a preferred decay channel would be needed.

Maiani, Polosa and Riquer (2016, 2018) impose additional selection rules to make the behaviors compatible with the diquark model and predict two-meson decay widths of order $1/N_c^3$ and $1/N_c^4$.

The particular case of the light meson scattering amplitudes has been thoroughly studied by Pelaez *et al.* (2004-2016) within the framework of chiral perturbation theory. The existence of scalar mesons with a tetraquark type structure has been found. In the large-N_c limit, their masses and decay widths diverge as $N_c^{1/2}$. Such behaviors do not fit the characteristics of compact tetraquarks, as conjectured by Weinberg. These scalar mesons would rather fit a molecular type structure.

Weak point of the large-N_c analysis

Contrary to the case of two-point functions of quark bilinear currents and their saturation by ordinary meson states, possible tetraquark contributions to two-meson scattering amplitudes are competing with the background of two-meson states, which saturate consistently the corresponding correlation functions. The hypothesis of an eventual absence of tetraquark states does not lead to any inconsistency.

Therefore, the predictions made for tetraquark decay amplitudes are based on the assumption of their possible existence and should be consisdered as upper bounds.

Line of approach

Study of possibly existing tetraquark properties through the analysis of meson-meson scattering amplitudes.

Four-point correlation functions of color-singlet quark bilinears,

$$j_{ab} = \overline{q}_a q_b,$$

having coupling with a meson M_{ab}:

$$\langle 0|j_{ab}|M_{ab}\rangle = f_{M_{ab}}; \quad f_M \sim N_c^{1/2}.$$

Spin and parity ignored; not relevant for the qualitative aspects.

Consider all possible channels where a tetraquark may be present.

To be sure that a QCD diagram may contain a tetraquark contribution, through a pole term, one has to check that it receives a four-quark contribution in its s-channel singularities, plus additional gluon singularities that do not modify the N_c-behavior of the diagram. Their existence is checked with the use of the Landau equations.

Diagrams that do not have s-channel singularities, or have only two-particle singularities (quark-antiquark), cannot contribute to the formation of tetraquarks at their N_c-leading order. They should not be taken into account for the N_c-behavior analysis of the tetraquark properties.

Exotic tetraquarks

Four distinct quark flavors, denoted 1,2,3,4, with meson currents

$$j_{12} = \overline{q}_1 q_2, \quad j_{34} = \overline{q}_3 q_4, \quad j_{14} = \overline{q}_1 q_4, \quad j_{32} = \overline{q}_3 q_2.$$

The following scattering processes are considered:

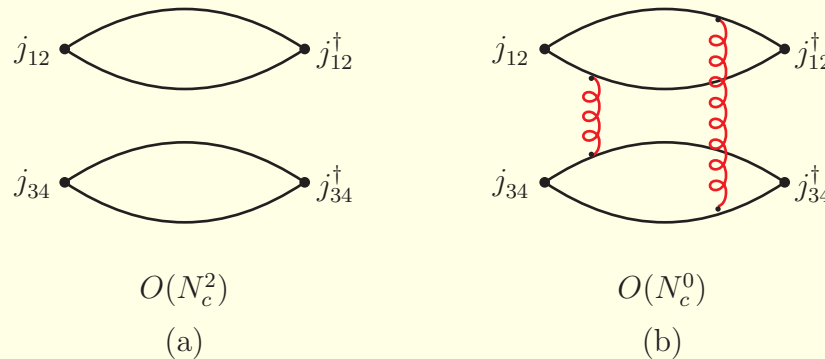$$M_{12} + M_{34} \ \to \ M_{12} + M_{34}; \quad \text{Direct channel I;}$$

$$M_{14} + M_{32} \ \to \ M_{14} + M_{32}; \quad \text{Direct channel II;}$$

$$M_{12} + M_{34} \ \to \ M_{14} + M_{32}; \quad \text{Recombination channel.}$$

'Direct' 4-point functions

$$\Gamma_I^{(\mathrm{dir})} = \langle j_{12} j_{34} j_{34}^\dagger j_{12}^\dagger \rangle \,, \qquad \Gamma_{II}^{(\mathrm{dir})} = \langle j_{14} j_{32} j_{32}^\dagger j_{14}^\dagger \rangle \,.$$

Leading and subleading diagrams for $\Gamma_I^{(dir)}$:

$O(N_c^2)$

(a)

$O(N_c^0)$

(b)

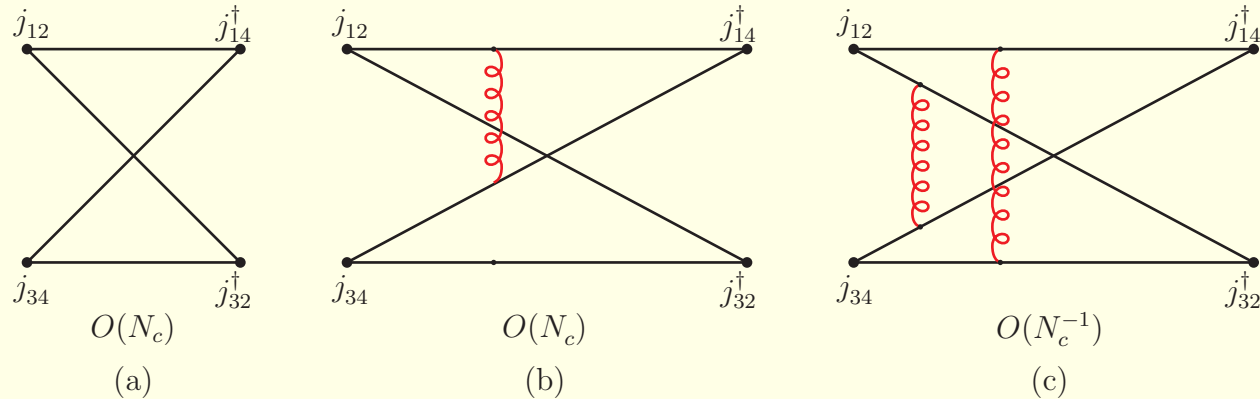Similar diagrams for $\Gamma_{II}^{(dir)}$.

Only diagram (b) may receive contributions from tetraquark states.

'Recombination' 4-point function

$$\Gamma^{(\mathrm{recomb})} = \langle j_{12} j_{34} j_{32}^\dagger j_{14}^\dagger \rangle \, .$$

Leading and subleading diagrams:

$$j_{12} \qquad\qquad j_{14}^\dagger \qquad\qquad j_{12} \qquad\qquad j_{14}^\dagger \qquad\qquad j_{12} \qquad\qquad j_{14}^\dagger$$

$$j_{34} \qquad\qquad j_{32}^\dagger \qquad\qquad j_{34} \qquad\qquad j_{32}^\dagger \qquad\qquad j_{34} \qquad\qquad j_{32}^\dagger$$

$$O(N_c) \qquad\qquad\qquad O(N_c) \qquad\qquad\qquad O(N_c^{-1})$$

(a)   (b)   (c)

Only diagram (c) may receive contributions from tetraquark states.

Direct and recombination amplitudes have different behaviors in N_c.

The solution requires the contribution of two different tetraquarks, T_A and T_B, each having different couplings to the meson pairs.

One finds for the tetraquark – two-meson transition amplitudes:

$$A(T_A \to M_{12}M_{34}) = O(N_c^{-1}), \qquad A(T_A \to M_{14}M_{32}) = O(N_c^{-2}),$$

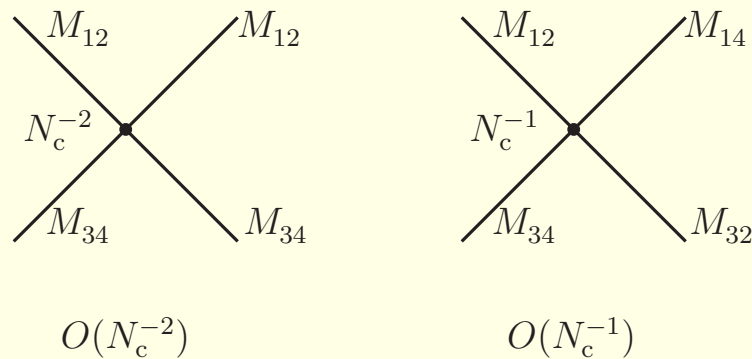$$A(T_B \to M_{12}M_{34}) = O(N_c^{-2}), \qquad A(T_B \to M_{14}M_{32}) = O(N_c^{-1}).$$

$$T_A \sim (\bar{q}_1 q_4)(\bar{q}_3 q_2), \qquad T_B \sim (\bar{q}_1 q_2)(\bar{q}_3 q_4).$$

Mixings of order $1/N_c$ between the two configurations are possible.
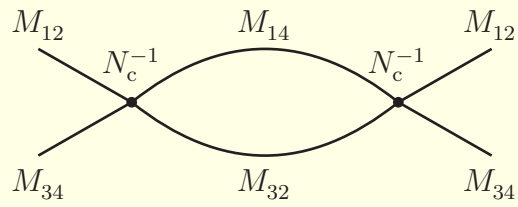
Favors a color singlet-singlet structure of the tetraquarks.

There is also generation of background meson-meson effective interaction by means of four-meson effective couplings. Recombination (quark exchange) effective couplings dominate.
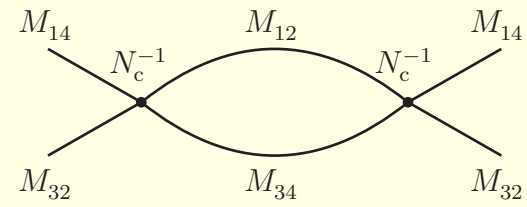
$$M_{12} \qquad M_{12} \qquad\qquad M_{12} \qquad M_{14}$$

$$N_{\mathrm{c}}^{-2} \qquad\qquad\qquad N_{\mathrm{c}}^{-1}$$

$$M_{34} \qquad M_{34} \qquad\qquad M_{34} \qquad M_{32}$$

$$O(N_{\mathrm{c}}^{-2}) \qquad\qquad O(N_{\mathrm{c}}^{-1})$$

These, in turn, generate meson loops.

Two-meson contributions.

M_{12}　　　　　M_{14}　　　　　M_{12}

N_c^{-1}　　　　N_c^{-1}

M_{34}　　　　　M_{32}　　　　　M_{34}

$$O(N_c^{-2})$$

M_{14}　　　　　M_{12}　　　　　M_{14}

N_c^{-1}　　　　N_c^{-1}

M_{32}　　　　　M_{34}　　　　　M_{32}

$$O(N_c^{-2})$$

M_{12}　　　　　M_{14}　　　　　M_{14}

N_c^{-1}　　　　N_c^{-2}

M_{34}　　　　　M_{32}　　　　　M_{32}

$$O(N_c^{-3})$$

M_{12}　　　　　M_{12}　　　　　M_{14}

N_c^{-2}　　　　N_c^{-1}

M_{34}　　　　　M_{34}　　　　　M_{32}

$$O(N_c^{-3})$$

M_{12}　　　　M_{14}　　　　M_{12}　　　　M_{14}

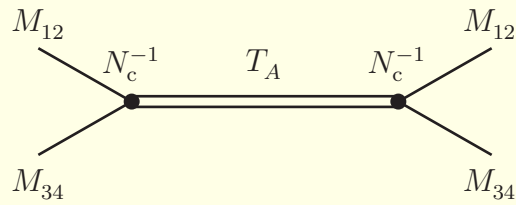N_c^{-1}　　　N_c^{-1}　　　N_c^{-1}

M_{34}　　　　M_{32}　　　　M_{34}　　　　M_{32}
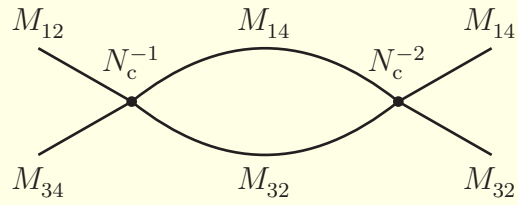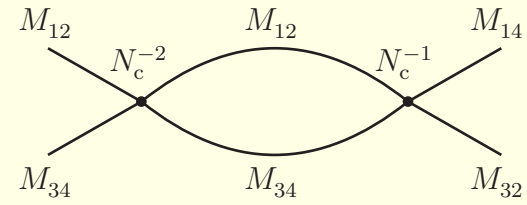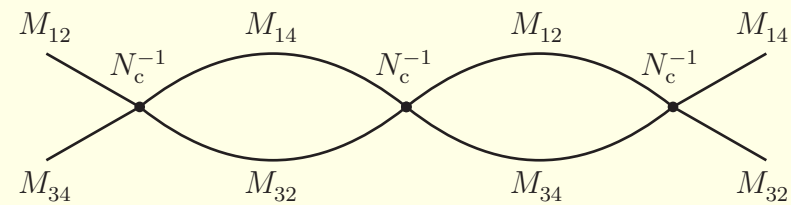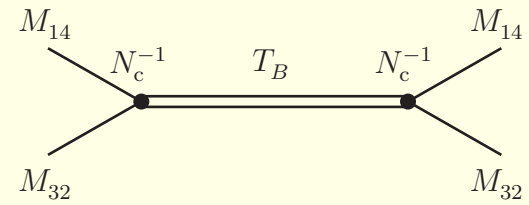
$$O(N_c^{-3})$$

Tetraquark intermediate states in meson-meson scattering.

$$M_{12} \qquad N_c^{-1} \qquad T_A \qquad N_c^{-1} \qquad M_{12}$$

$$M_{34} \qquad\qquad\qquad\qquad\qquad\qquad M_{34}$$

$$O(N_c^{-2})$$

$$M_{14} \qquad N_c^{-1} \qquad T_B \qquad N_c^{-1} \qquad M_{14}$$

$$M_{32} \qquad\qquad\qquad\qquad\qquad\qquad M_{32}$$

$$O(N_c^{-2})$$

$$M_{12} \qquad N_c^{-1} \qquad T_A \qquad N_c^{-2} \qquad M_{14}$$

$$M_{34} \qquad\qquad\qquad\qquad\qquad\qquad M_{32}$$

$$O(N_c^{-3})$$

$$M_{12} \qquad N_c^{-2} \qquad T_B \qquad N_c^{-1} \qquad M_{14}$$
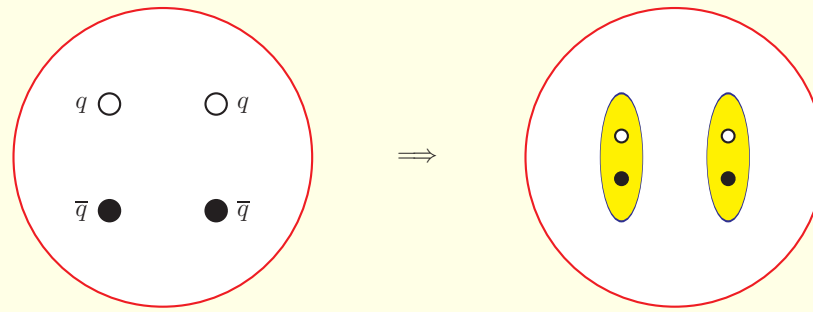
$$M_{34} \qquad\qquad\qquad\qquad\qquad\qquad M_{32}$$
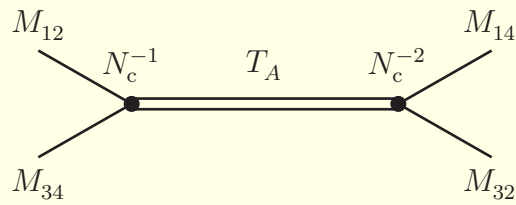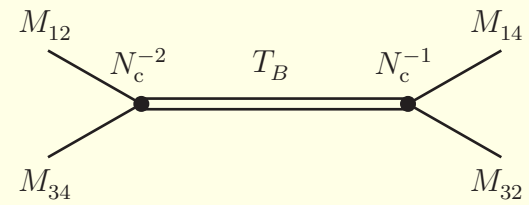
$$O(N_c^{-3})$$

The fact that we have two different tetraquarks, each having a structure made of two color-singlet clusters, raises several questions.

First, once the color-singlet clusters are formed inside the four-body system, their mutual interaction can no longer be confining.


At most, one might expect long-range type Van der Waals forces, reminiscent of the confining forces. In that case, the tetraquarks, if they exist, would be loosely bound, very similar to molecular type tetraquarks.

This possibility has already been foreseen by Jaffe (2008).

Second, the necessity of having two different tetraquarks to accomodate the N_c-counting constraints does not fit the diquark formation scheme, where only one type of diquark is expected to be formed, in its color-antisymmetric representation.

The diquark formation scheme:
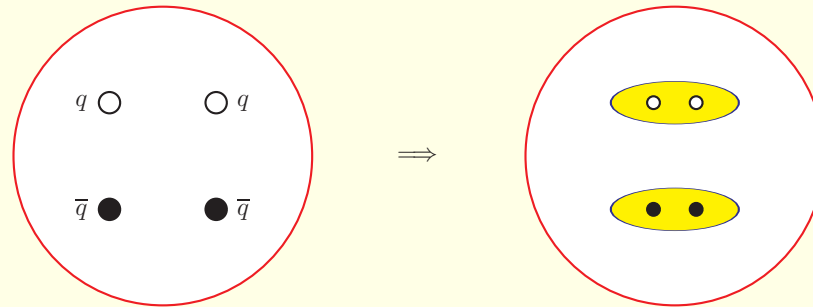
The binding of the diquark and antidiquark clusters is expected to be realized by means of confining forces, hence the appearance of a compact tetraquark.

To make the diquark scheme compatible with the N_c-analysis, Maiani, Polosa and Riquer (2018) impose additional constraints for the selection of diagrams contributing to the formation of tetraquarks.

- Only non-planar diagrams contribute. This lowers the contribution of direct channel diagrams by two degrees in N_c.
- To equate the N_c-degrees of direct and recombination channel contributions, only diagrams with quark loops in recombination channels are accepted.

The above constraints allow the presence of only one tetraquark, with two-meson decay widths of the order of $1/N_c^4$.

The second constraint, while mathematically correct, remains obscure at the physical level.

Conclusion

The large-N_c limit of QCD allows us to have a complementary insight into the problem of multiquark states.

Tetraquark and multiquark states, if they exist, do not generally appear in N_c-leading-order terms and are competing with multimeson background contributions. The conventional large-N_c-based analysis does not lead to a proof of their existence, but simply gives upper bounds for their decay or transition amplitudes.

The generic results, in the case of four quark flavors, have the tendancy to favor the formation of tetraquarks with color-singlet internal clusters. In such a case, the tetraquarks would probably be loosely bound.

The diquark scheme, which might lead to the emergence of compact tetraquarks, requires fine tuning dynamical mechanisms.

Resolution of four-body bound state equations and lattice calculations may be of great help for a better understanding of the question.